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On the Progress to Accuracy of Logarithmic Tables.

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At the meeting of the Society on June 14 (*Notices*, vol. xxxii. p. 290), in the course of some remarks on the original logarithmic tables, I stated that it was my intention to form at once a complete list of the errors in Vlacq's table of 1628 that would affect a seven-figure table of the logarithms of numbers, with the view of seeing whether any of them were still reproduced in the modern tables. After what has appeared on the subject (*Notices*, vol. xxxii. pp. 255, &c., and pp. 288, &c., May and June, 1872), it is only necessary here to recall to mind that the first table of decimal logarithms published was Briggs's *Logarithmorum Chilias Prima*, London, 1617, containing the logarithms of only the first thousand numbers to 14 decimals. This was followed by Briggs's *Arithmetica Logarithmica*, London, 1624, containing the logarithms of the numbers from 1 to 20,000, and from 70,000 to 90,000 to 14 decimals; and in 1628 Vlacq filled in the gap of 70,000, and published at Gouda his *Arithmetica Logarithmica*, giving the logarithms of the numbers from 1 to 100,000 to 10 decimals. This last work was also published with an English introduction by one George Miller, London, 1631.

Vlacq's work of 1628 being the first that gave the logarithms of the unbroken series of numbers from 1 to 100,000, is the original from which every complete table has been copied, either directly or indirectly, and after more or less revision, for (with the exception of the French manuscript tables that have never been published) no fresh calculation has been made. In vol. xxxii. p. 255, of the *Notices*, I have given references to all the places where, as far as I could find, errors in Vlacq had been published, and certain of these are reprinted there, so that that paper, taken together with Vega's and Lefort's lists, gives very likely all the known errors. As, however, Vlacq's is still the most convenient and accessible ten-figure table, and (with the exception of Vega, 1794) is unique, it would be desirable to collect in one place the errata from every source, and print them all together in one place; but such a list, containing over 600 errata, would occupy so much space, and be used by so few, that the advisability of publishing it, at all events in the *Notices* of this Society, seems questionable. This objection, however, does not apply to the errors which would affect a seven-figure table, and which, though considerably more numerous than I anticipated, amount to only 171, less than one-third of the total number. My object in making the list originally was to see if all the errors were corrected in the seven-figure tables now in use, and if not, to form separate and subsidiary lists for each of the latter, supplemented by any others that I could collect from other sources. This intention was modified for two reasons; first, because it appeared, on comparison, that the best modern tables are quite

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free from the Vlacq's errors; and secondly, because the places in which errors in logarithmic tables have been published are so numerous and so scattered that it would require a considerable amount of research, and occupy much time to treat the subject with only tolerable completeness. The latter fact appeared in the course of the preparation of the 1872 report of the Mathematical Tables Committee of the British Association, in which an attempt was made to give, under each work described, references to the various journals, &c., where errata in it had appeared; but even this limited proposal could not, for several reasons, be carried out satisfactorily. The Committee, however, intend to make the formation and publication of as complete lists as can be obtained of errata in mathematical tables now in use (whether logarithmic or not), a prominent feature in a future report; and this will supersede the partial performance of the same work which I originally intended to give here. The results, therefore, contained in this note are not presented as having more than slight practical importance, their chief claim to notice being the historic interest attaching to the subject. They are to be regarded in the following light:—A table is published containing a certain number of errors (miscalculations, misprints, &c.); it is greatly and extensively used in all the sciences, and repeatedly reprinted during two centuries and a half, though never recalculated. The original errors thus become gradually eliminated, and it is a matter of a good deal of interest to watch the progress of accuracy with the lapse of time, and see how long the struggle lasted before truth ultimately gained the complete victory.

It will be seen that there is an interval of more than 200 years between the original publication and the appearance of the first table that is quite free from the hereditary Vlacq's errors, and that even now the triumph of accuracy is not quite perfect, as the seven-figure table last published in England (1871) contains two of the original errors.

The following is a complete list of all the errors in Vlacq's ten-figure table of 1628 that affect the first seven decimals of the logarithms, the seventh figure being increased if the succeeding figures are greater than 500. I formed the table myself by a careful comparison of Vlacq with the errata lists referred to in the paper in the *Notices*, to which reference has been made above, and I examined it carefully after it was formed, so that I believe it to be quite complete and correct. It contains 171 errata, but of these 77 were given by Vlacq himself after the preface to the *Arithmetica Logarithmica*, so that he can only fairly be charged with 94. I have thought it better, however, to include all in the list, so as to make it complete, but have denoted those which Vlacq himself pointed out, by an asterisk. The propriety of including these is strengthened by the fact that Vlacq's errata list was not reproduced in the English copies of 1631—an omission which shows the extreme carelessness with

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which that edition was prepared. As a consequence, many of Vlacq's errors, that he corrected himself, are reproduced by John Newton, 1658, who could not have seen the Gouda edition of 1628.

It is to be noted, that though I have spoken throughout of *Vlacq's* Errors, several of them occurring in the portions of the table from 1 to 20,000, and from 90,000 to 100,000, were reproduced from Briggs, 1624.

List of Errata in Vlacq's "Arithmetica Lotharithmica," Gouda, 1628, *that affect the first seven decimals of the Logarithms of the numbers from* 1 *to* 100,000.

No.	Error.	Correction.
* 80	9030894	9030900
* 169	2278868	2278867
* 183	2024511	2624511
* 238	3765769	3765770
* 580	7634180	7634280
* 590	7798520	7708520
*1239	0930712	0930713
*1298	1132746	1132747
*1309	1169336	1169396
*1321	1209023	1209028
*1354	1316177	1316187
*1359	1332175	1332195
*1377	1389334	1389339
*1626	2111203	2111205
*2167	3358579	3358589
*2434	3863204	3863206
*2534	4038076	4038066
*2544	4055161	4055171
*3329	5223128	5223138
*3499	5439449	5439439
*4599	6626644	6626634
*4906	6907295	6907275
*5126	7097736	7097786
*5194	7155029	7155019
*5222	7178339	7178369
*6207	7928811	7928817
*6257	7663662	7963662
*6841	8351106	8351196
*6941	8414230	8414220
*6957	8424320	8424220
7392	8667620	8687620
*7775	3907004	8907004
*7830	2937618	8937618
8172	9123234	9123284
*8556	8322708	9322708
8688	9389193	9389198
*8692	9391147	9391197
*8832	9460541	9460591
*9174	9925587	9625587
*9176	9626524	9626534
9182	9029373	9629373
*9317	9602761	9692761
*9354	9709973	9709974
9429	9644656	9744656
*9480	9798083	9768083
*9482	6769000	9769000
*9972	9987833	9987823
*9973	9988263	9988258
*10058	0025166	0025116
*10061	0026413	0026411
*10096	0041443	0041493
10822	0333075	0343075
*10847	0353006	0353096
*10859	0357808	0357898
*11440	0584269	0584260
*11469	9595256	0595256
*11920	9762763	0762763
11954	5775133	0775133
*11955	0775495	0775496
12218	0970001	0870001
*12328	0908929	0908926
*12398	0933566	0933516

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No.	Error.	Correction.	No.	Error.	Correction.
*13274	1230016	1230018	41490	6179439	6179434
*14020	1467481	1467480	42344	6267909	6267919
*14527	1621754	1621759	42506	6284505	6284502
*14763	1691749	1691746	44026	6437042	6437092
*14786	1698501	1698507	44656	6498748	6498798
*15843	1998334	1998374	48033	6815367	6815397
*16461	2164592	2164562	48376	6846267	6846300
*17509	2432623	2432613	48764	6880992	6880993
*17773	2497807	2497607	49328	6931935	6930935
*19009	2789543	2789593	49502	6946237	6946227
19087	2087377	2807377	49717	6965059	6965049
19088	2087604	2807604	*49880	6979284	6979264
*19107	2811923	2811925	50479	7031105	7031107
*19113	2813239	2813289	50507	7033506	7033516
*19195	2831861	2831881	50897	7086922	7066922
19315	2758947	2858947	51193	7092108	7092106
20832	3188310	3187310	52943	7238086	7238085
23806	3766894	3766864	53919	7317419	7317418
23999	3401931	3801931	54033	7326541	7326591
24626	3913940	3913939	55692	7457828	7457928
24862	3955381	3955361	56832	7545925	7545929
27164	4339935	4339937	57628	7606336	7606335
*28423	4536299	4536699	57629	7606410	7606411
28758	4587547	4587587	57941	7629856	7629860
29282	4666017	4666007	59502	7745314	7745316
30972	4909683	4909693	59838	7769781	7769771
33071	5194474	5194473	*60400	7810364	7810369
*33800	5280167	5289167	60844	7842173	7842178
33832	5292277	5293277	61163	7864848	7864888
34259	5347757	5347747	61872	7914972	7914942
34728	5406898	5406798	61984	7922786	7922796
*36560	5630082	5630062	61999	7923947	7923847
36935	5674361	5674381	62090	7930215	7930217
37091	5692615	5692685	62267	7942575	7942579
38321	5834568	5834368	62681	7971358	7971359
*38780	5886018	5886078	62759	7976770	7976760
38889	5898298	5898268	63688	8040596	8040576
38962	5906413	5906412	63747	8044597	8044598
39844	6003429	6003629	64125	8070374	8070274
39845	6003638	6003738	64183	8074100	8074200
40403	6064126	6064136	64445	8091992	8091892
*40598	6080546	6085046	64953	8125962	8125992
41018	6129795	6129745	65097	8135510	8135610
41407	6170728	6170738	65217	8143508	8143608

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No.	Error.	Correction.	No	Error.	Correction.
65537	8164876	8164866	77662	8902046	8902086
66607	8235299	8235199	*78700	8959748	8959747
66759	8245198	8245098	80212	9042392	9042393
*67050	8263688	8263988	80554	9060831	9060871
67951	8321958	8321959	81674	9120837	9120838
69318	8408450	8408460	86897	9389948	9390048
69579	8424762	8424782	97105	9872418	9872416
73653	8671804	8671904	97828	9904532	9904632
74703	8733480	8733380	98921	9952865	9952885
74742	8735642	8735647	*99090	9960294	9960298
74792	8738557	8738551			

The first thing that strikes any one on looking at this list is the great number of errata that are clearly only misprints—some of them very gross; thus in log. 11469 and log 11920 the first figure is printed a 9 instead of a 0. Such errors as these no amount of carelessness on the part of an editor could reproduce, as they could not occur in seven-figure tables, arranged in the manner now universal. Another point that attracts attention is the great number of errors between 1 and 10,000, all of which must be mere misprints, as they are not reproduced in the logarithms of the corresponding numbers multiplied by 10, that occur in the portion of the table between 10,000 and 100,000. Vlacq's table is so far exceptional that it proceeds from 1 to 100,000, while all its successors, which extend to 100,000 (except Roe, 1633), commence at 10,000, or thereabouts; it is proper, therefore, in making comparisons, to subtract from Vlacq's total of 171 the number of errors occurring in the first ten thousand logarithms (48), and we have 123 left. Errors in Vlacq's differences have not, of course, been noticed. It is, perhaps, worth while to give the errors in log. 80, which is printed 9030893870 instead of 9030899870, as in all the copies but one that I have seen the correction has been carefully made with the pen, so as to render it impossible to see how the logarithm was originally printed.

The following is a list of seven-figure tables (extending from 10,000 to 100,000), arranged in order of date, together with the number of the above-mentioned 123 Vlacq's errors that are reproduced in each. The list has no claim whatever to completeness; it merely is formed of the books satisfying the requisite conditions that I could conveniently lay my hands upon. There will not be found, however, I think, any important omission, as all the leading works which form the main line of descent from Vlacq to the present day are included, and the tables left out consist chiefly of unimportant foreign reprints, which (though the addition of them would have afforded interesting results) were not necessary for the purpose I had in view:—

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Vlacq, Arithmetica Logarithmica, Gouda, 1628, fol. (ten decimals), and Logarithmicall Arithmetike, London, 1631, fol. (ten decimals)	123
John Newton,* Trigonometria Britanica, London, 1658, fol. (eight decimals)	98
Sherwin, Mathematical Tables, London, 1706. 8vo,†	65
„ „ 2nd edition, London, 1726, 8vo	58
„ „ 3rd edition „ 1741 „	20
Gardiner, Tables of Logarithms, London, 1742, 4to	19
„ Tables de Logarithmes, Avignon, 1770, 4to (edited by Pezenas, Blanchard, and Dumas)	17
Sherwin, Mathematical Tables, 5th edition, London, 1771, 8vo (edited by Samuel Clark)	20
Hutton, Mathematical Tables, London, 1785, 8vo	12
Taylor, Tables of Logarithms, London, 1792, large 4to	6
Callet, Tables Portatives de Logarithmes, Paris, 1793, 8vo	11
Vega, Thesaurus Logarithmorum completus, Leipzig, 1794, fol. (ten decimals)	23
Hutton, Mathematical Tables, London, 1794. 8vo	10
Vega, Tabulæ Logarithmo-trigonometricæ. edit. secunda, Leipzig, 1797, 2 vols. 8vo	5
Hutton, Mathematical Tables, 3rd edit. London, 1801, 8vo	5
„ „ 6th edit. London, 1822 „	5
Babbage, Table of Logarithms, London, 1827, 8vo	1
Hassler, Tabulæ Logarithmicæ, New York, 1830, 12mo. (also with English, French, &c. titles)	2
Babbage, Tables of Logarithms, London, 1831, 8vo	1
Hülsse, Vega's Sammlung Mathematischer Tafeln, Leipzig, 1840, 8vo	1
Babbage, Table of Logarithms, London, 1841, 8vo	1
Shortrede,† Logarithmic Tables, Edinburgh, 1844, large 8vo	7
Bell, Mathematical Tables (Chambers' Educational Course), Edinburgh, 1847, 8vo	6
Shortrede,‡ Logarithmic Tables, Edinburgh, 1849, 2 vols. large 8vo	6
Anonymous, Mathematical Tables (Chambers' Educational Course), Edinburgh, 1853, 8vo	6
Hutton, Mathematical Tables, London, 1855, 8vo (edited by Olinthus Gregory)	4
Callet, Tables de Logarithmes, Paris, 1855, 8vo	2
Bremiker, Vega's Logarithmic Tables (translated from the fortieth edition of Dr. Bremiker's, by W. L. F. Fischer), Berlin, 1857, 8vo	0
Schrön, Siebenstellige gemeine Logarithmen. Braunschweig, 1860, large 8vo	0
Callet, Tables de Logarithmes, Paris, 1862, 8vo (Revue par J. Dupuis)	0

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* This table gives the logarithms to eight decimals; in making the comparison it was treated as a seven-figure table (last figure corrected), care being taken to give it the benefit of the doubt if the eighth decimal was a 5.

† In this list the terms '8vo.' and '4to.' are used merely to convey an idea of the size of the work, without regard to the mode of printing. Thus Shortrede, Bruhns, and Sang, are nearly exactly the same size, although, as far as the printing is concerned, the first two are 4to, and the last 8vo. A similar remark applies to the editions of Hutton, the earlier ones being 4to, and the more recent 8vo, although the size is the same. Except for purely bibliographical purposes, the description of a book as 8vo, 4to—these words being used in their technical sense—is at all events useless, and may be worse, misleading.

‡ Shortrede commences at 10.800, but this makes no difference in the comparison with other works, as none of the more recent tables contain any Vlacq's errors between 10,000 and 12,000.

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Bruhns, a New Manual of Logarithms, Leipzig, 1870, large 8vo	0
Sang, a New Table of Seven-place Logarithms, London, 1871, large 8vo	2

The comparisons were only made for portions of the table, from 10,000 to 100,000. In most works there is a separate table of the logarithms of the first 1000 or 1200 numbers prefixed; but these are, no doubt, free from (hereditary) error.

The above list shows the progress to accuracy of logarithmic tables; of Vlacq's 123 errors he corrected 34 in the errata-list prefixed to his work. Of this list, which did not appear in the English copies of 1631, John Newton was not aware, so that, though apparently correcting 25 of the errors, his table was really retrograde as far as accuracy was concerned, for it contains 98 errors, against 89 only which had escaped Vlacq. I must, however, here state that in counting the errors in any work, I have in all cases *ignored any errata-list*, whether occurring in the work itself or elsewhere. The adoption of this rule (though it presses very unfairly on one or two editors who revised their works after printing, and before publication) was necessary, as a page of errata was often introduced into only the later portion of an impression, or sometimes the errata-list was continually amplified and reset during the making up from time to time of the copies in the same impression. The numbers, therefore, in all cases refer to the number of Vlacq's errors in the table *as printed*, and consequently Vlacq's own number is given as 123. Sherwin, it will be remarked, made improvements in his first and second editions, but the great additional accuracy in the third is due to Gardiner, who carefully revised the work, and published his own table in the succeeding year.

In regard to the next succeeding tables, the most remarkable facts are the near approach to accuracy made by Michael Taylor in 1792, and the great number of errors in Vega, 1794; the latter result is most curious, as Vega is generally regarded (and justly) as the great purifier of the table of logarithms of numbers, and the sole reason why he makes so discreditable an appearance in the above list is the non-recognition of his own errata-lists. (See *Notices*, vol. xxxii. p. 288.) In point of fact every error affecting the first seven places was known to Vega; as in the more complete of his two errata-lists he gives the corrections to *all* the 23 errors that occur in the tabular portion of his work, though, through the omission of several asterisks that ought to have been inserted, a casual reader might imagine that such was not the case. Vega thus in (or soon after) 1794 was acquainted with every error affecting a seven-figure table, and he was the first to be in that position. It is most strange that the editor who detected *every* (seven-place) error should appear in the list as the very worst offender of all. Taylor left uncorrected only six errors, viz. in the logarithms of 38962, 52943, 57628, 57629, 63747, 67951; and of these

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the last five are correctly given by Vega in his tables, so that Vega's *tables* (without the errata-list) and Taylor taken together leave only one error, which Vega speedily found out. Thus by about 1794 all the errata were known, and Babbage in 1827 only by a remarkable oversight failed to eliminate them all from his table. The one that occurs is that in log 52943;* how this could have escaped detection is most difficult to conceive, as Babbage's table was twice read with the Vega of 1794, in which the logarithm is correctly given. Bremiker's Vega, 1857, is the first work in the list (though, doubtless, not quite the first published), which is free in the table itself from hereditary error. It is well known that Babbage had access for the purpose of comparison to the French manuscript tables, and but for the present investigation, it might be thought that he was enabled to obtain so near an approach to accuracy by their means. Such, however, is in nowise the case, and seven-figure logarithms owe none of their accuracy to the French tables.

The two errors, which, though discovered by 1794, exhibit such a persistent vitality, are those in log 38962 and log 52943, and they occur in all the tables, subsequent to 1827, where two or more are assigned in the list. It was only so recently as the present year that Mr. Sang wrote to the *Athenæum* (June 8, 1872) about the error in log 52943, which had been pointed out to him as occurring in his table, and which he regarded as a fresh discovery.

It is to be clearly understood that the list shows only the numbers of Vlacq's errors that are reproduced in each of the works named; and does not convey any information with regard to the total number of errors in any of these works; it represents, so to speak, the editor's knowledge and research, but not his, or the printer's care in revising the proofs. Thus besides the numbers given there might, as far as the list is concerned, be any number of fresh errata introduced by careless copying and revision, or by false corrections, viz. alterations from right to wrong. Thus the Sherwin of 1771, Hutton speaks of as the most inaccurate table he ever saw, but this is not apparent from the list; the table is inaccurate, if it be so (and I think it likely that it is) through careless press revision, and a mere cursory examination of the pages shows that not sufficient care has been taken.

With regard to the way in which the tables were compared, I need only say that all the logarithms in the list of Vlacq's errors were examined with each work, and marked according as they were corrected or no. Some I did myself, but the majority were carefully performed by an assistant, who went through the

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* A still more curious fact with regard to log 52943 is that although it is correctly given by Vega in his *Thesaurus* (ten decimals) 1794, and in his *Tabulæ* (seven decimals) 1797; it was altered from right to wrong by Hülsse in his edition of Vega (seven decimals) 1840, where it appears as of old, before Vega's correction.

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work twice: whenever the results appeared to be curious I examined the table myself and always found it correct; the numbers given may therefore, I think, be relied on. In several cases errors were found in the logarithms which were wrong in Vlacq, but in which Vlacq's error was not exactly reproduced. When the error was a misprint, clearly independent of Vlacq, it was not counted, but when it was obviously a partial correction, and had its origin in Vlacq's error it was included. To any one who knows how great has been the number of logarithmic tables that have been published, the list given above may seem meagre, but it is to be remembered that the works included were subjected to a double condition, viz. that they were to give seven decimals and extend to 100,000. This greatly limits the number of works, as very many give only five or six decimals, or extend only from 1000 to 10,000. The complete seven-figure tables form the grand line of succession; and, as I said, the list is fairly perfect; most of the numerous 8vo editions of Vega and Callet are omitted, but they belong to comparatively recent times, and are not of very much importance. The chief succession is Briggs, Vlacq, Roe, Newton, Sherwin, Gardiner, and then there are two branches, viz. Hutton founded on Sherwin, and Callet on Gardiner, and an independent shoot, Vega. I have not included in the list Nathaniel Roe, who published his *Two Tables of Logarithmes* in 1633, and was the first promulgator of a complete seven-figure table, as from his not having corrected the seventh decimal when the succeeding figures were greater than 500, his table cannot properly be compared with the others without a good deal of trouble. The comparison with the Vlacq's errata-list assigns him 3 errors between 1 and 10,000, and 61 between 10,000 and 100,000 (the table extends from 1 to 100,000); but these numbers are not to be relied on at all, even as given a rough estimate of accuracy, as in obtaining them a disagreement was counted as an error or not, according to whether it seemed likely that the discrepancy was due to an error or the want of contraction of the last figure, the benefit of the doubt being nearly always given to Roe. It did not seem worth while making a list of Vlacq's errors to more decimals, for the sake of this one table alone. Roe made the first step towards the arrangement of seven-figure tables as at present, viz. he contracted the numbers by making the last two figures 01,02..50, or 51..99 serve for the whole line, and the logarithms by making the first two or three figures serve for the whole column, and Newton completed the arrangement by making the leading figures of the logarithm common to the block. All things considered, in spite of Vlacq, Vega, Wolfram, and others, it seems to me that this country has throughout fairly kept the lead in logarithmic calculations that it acquired originally by the labours of Napier, Briggs and Gunter.

I have not yet referred to what appears to me to be one of the most interesting points on which the results in this paper throw light, viz. the influence of a perfect 'free trade' in tables. Nothing

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could have been left more completely to private enterprise than the publication of logarithmic tables, and the consequence has been that errors have continually reappeared long after their discovery and publication. Had any National Observatory or other Government office, or Academy in any country published tables with their authority, and received and corrected errors as they were pointed out, a perfectly correct table would have no doubt appeared much earlier, and the continual reproduction of inaccuracies would have been avoided. As a matter of fact, the last half-dozen errors that survived were really of no practical consequence, as no one relies on the last figure of a result obtained by logarithmic calculations; but what the user of a table wants is to be *certain* that it is free from error. As a rule, no one would edit a logarithmic table, unless he considered himself more or less competent to do so; but the list of errors shows that many who have published such tables were not properly qualified by their knowledge of what had been previously effected with regard to the attainment of accuracy. Take, for example, the case of the late General Shortrede whose reputation as a calculator stands, most deservedly, very high; his table contains errors not from any want of care in examination, no doubt, but simply because he was not aware of the necessity of making an investigation similar to that contained in this paper, or perhaps was unable (by his residence in India or for some other cause) to do so. The same remark also applies to Mr. Sang and to every one who has published a table containing an error, subsequent to 1794; and really also (though the facts in this paper do not render it so apparent) to all the editors of tables previously to that date. Now, had there been any body, such as the Greenwich Observatory or the late Board of Longitude, which would have officially issued a table of logarithms, and undertaken to print every year in their Observations or other periodical publications the errors discovered, besides correcting them in subsequent editions, the amount of accuracy attained, and, what is more important, the amount of confidence felt, in such a work would have been far in excess of what absolutely has been the case. I only advocate a national observatory or other government establishment undertaking such a work, on account of their permanence. A table published by an individual cannot well hold its position for more than forty years for many reasons: Callet and Hutton seem exceptions, but they have been in the hands of great publishing houses which have employed fresh editors from time to time to continually revise them, more or less: they have done, in fact, what I say some government office (or other permanent institution) ought to have done for the last 200 years. Tables, as originally calculated, always *will* have errors which can only be found out one by one in process of time, and I think it is clear that if there is no permanent body which is charged with the reception and corrections of errors as they are found out they will continually be reproduced through the want of knowledge on

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the part of editors. Such a superintendence would not in any way infringe on private rights, any more than the publication of the *Nautical Almanac* does. It is to be presumed that if the latter were not published by the State some one would think it worth while to prepare such an almanac, but still no one would regard such a state of things as desirable. Logarithmic tables, besides their continual use in observatories, are of paramount importance in the solution of the triangles in trigonometrical surveys, in fact, Babbage's table was published for use in the calculations connected with the survey of Ireland; so that it is remarkable that it seems never to have occurred to any Astronomer Royal, or Commissioner* of Longitude, that mathematical tables were well worthy the attention of his departments.

All the above remarks apply with even greater force to collections of nautical tables, and the fact that there are not, I believe, any recognised tables published by the Admiralty, and for which it is responsible, is curious. In the course of the preparation of the British Association Report, I examined about twenty collections of nautical tables that have been published during the present century, by naval officers, proprietors of nautical academies, opticians, &c. It is to be inferred that all of these have been more or less used, but I do not see what guarantee of accuracy the purchasers of any could have had; at all events, in the absence of any strong reason to feel confidence in the author, I should not myself like to use the ordinary mathematical tables contained in most of them for calculations. That errors were not very uncommon is evidenced by such phrases as "second edition, with many errors corrected," &c., though, as the publisher's object is to sell his second edition, not to make the first equal in value to the second, there is usually a scrupulous avoidance of any reference as to where the errors occurred. Mrs. Taylor, in the preface to her *Luni-Solar and Horary Tables*, remarks that "some errors have crept into the calculations on account of the multiplicity of entries," for which she claims indulgence on the somewhat singular ground "that the system on which she has worked is mathematically correct and founded on sound principles;" but this naïve confession is, no doubt, pretty well unique, though I dare say the same cannot be said for the necessity thereof. What the collection of tables now in use in the Royal Navy may be I do not know (Raper, I should suppose; Hamilton Moore was the book half a century ago); but be it what it may, I suppose some pains have previously been taken to ensure its accuracy, and if so, it is a pity there is no generally accessible record of such revision having been made. Fresh tables are

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* The Board of Longitude seems to have taken no pains to ensure freedom from error or to facilitate the detection or correction of errors, even in works published by themselves as *e.g.* Hutton's *Powers of Numbers*. Errors in logarithms were published in different *Nautical Almanacs*, but there was no systematic attempt made on the part of the Board to render their tables accurate, or even in fact generally useful or known.

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continually being published, so that it is clear there is a demand for them; of course, there are different methods of determining the longitude, &c. (and the observational tables need constant improvement), so that one work only would not suffice. All, however, that I wish to point out is, the unsatisfactory nature of the past and present state of affairs with regard to tables for which there is a constant and general demand.* A *permanent* body that would charge itself with the reception and publication of errata in well-known tables would do much; but if such a body would, in addition, itself stereotype and publish the standard tables, continually correcting the plates, so that each edition would become more and more accurate, the errors in the earlier editions being noticed in the introduction, the result would be an amount of accuracy of the tables, and confidence in them on the part of calculators, such as has never yet been experienced, except in isolated cases.† Absolute accuracy is so essential for a table that a conscientious calculator will prefer to perform his computations *ab initio* rather than use a table, of the history, &c. of which he is ignorant; and it seems to me that only by means of some such body as the *Nautical Almanac* office or the Royal Society, is absolute accuracy, and the certainty that such is the case, attainable.

It is, perhaps, too late to do very much for tables of seven-figure logarithms, as it is pretty certain that the later editions of Babbage, Schrön, &c., are free from error, but still it would be something to keep future editors from reproducing obsolete errors; and there are still many tables, viz., ten-figure logarithms of numbers and trigonometrical functions, the natural trigonometrical canon, tables of multiplication, quarter squares, powers, &c., the accuracy of which is very far from being assured. For the next few years the British Association Committee on Mathematical Tables will do what they can with the view of obtaining as much accuracy as possible in tables relating to all branches of mathematics; but the Committee will only exist for a time, and the advantages of a continual supervision will not be in any way obtained.

A list, formed as before, is given below of the Vlacq's errors that occur in several seven-figure tables, the extent of which is generally from 1 to 10,000. Properly Briggs' *Arithmetica*, 1624, is the standard with which such tables ought to be compared; but I have not formed a complete list of errors in Briggs similar

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* The high mathematical tables are in one respect better off than the common ones; for, as they cannot but be a pecuniary loss to their authors or editors, no one thinks of publishing one who is not interested in the subject for its own sake, and really earnest in the cause of truth and accuracy: but profit can be made by the sale of the general tables, and they are published *to sell.*

† Barlow's tables (the L. U. K. edition) is a case in point; but it is not every one who would wish to employ the work that is aware of its accuracy. The first logarithmic table stereotyped was, I believe, the 1793 edition of Callet, but I say so only from an impression, not after investigation.

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to that given for Vlacq. As this very considerably diminishes the value of the list, I have made no attempt to complete or extend it, merely including, in fact, works I happened to have at hand to give an idea of the general state of care taken in the production of the small tables. Of Vlacq's 48 errors, he only derived 11 from Briggs: this is a confirmation of what was previously remarked, viz., that all the errors in the earlier portion of his table were merely the result of careless copying and revision.

Briggs, Arithmetica logarithmica, London, 1624, fol., 1 to 20,000 and 90,000 to 100,000 (fourteen decimals),	11
Henrion,* Traicté des Logarithmes, Paris, 1626, 8vo (1 to 20,000 fourteen decimals)	8
Brigge† [Briggs], Tables des Logarithmes, Gouda, 1626, 8vo (1 to 10,000)	11
Vlacq, Arithmetica logarithmica, Gouda, 1628, fol. (the portion from 1 to 10,000, ten decimals)	48
Norwood,‡ Trigonometrie, 2nd edition, London, 1651, 8vo (1 to 10,000)	2
Oughtred, Trigonometrie, London 1657, 8vo (1 to 10,000)	1
Gunter,§ Works of Edmund Gunter, 5th edition, 4to. London, 1673, small 4to (1 to 10,000)	3
Vlacq, Tabulæ Sinuum, &c., Amstelædami, 1681, small 8vo (1 to 10,000)	1
Vlacq, Tabellen der Sinuum, &c., Frankfort and Leipzig, 1757, small 8vo (1 to 10,000)	1
Donn, Mathematical Tables, 3rd edition, London, 1789, 8vo (1 to 10,000)	5
Bagay, Nouvelles Tables Astronomiques, Paris, 1829, small 4to (1 to 21,600)	1

All these tables, where it is not otherwise stated, give the logarithms to seven decimals.

De Morgan (in his article on Tables in the *English Cyclopædia*, 1861) makes a suggestion which he thinks may be worth the attention of future compilers, viz. to classify logarithmic tables according as to whether they came from Vlacq or direct from Briggs; and also with regard to their form (number of decimals, &c.), so that they might be divided into sets, those in each set being lineal descendants of their predecessors. I scarcely think it would be possible to carry out this proposal, as some editors (who seem to have imagined all tables must be accurate) have gone back direct to Vlacq and Briggs, while others have merely copied some table with which they happened to be acquainted. A few have also based their tables on a comparison of several. A classification according to sources could only be made by a careful investigation of the errors in each table; but it would scarcely,

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* On this work see *Phil. Mag. Supp.*, Dec. 1872 (S. 4. t. xliv. pp. 500 &c.)

† This table is the same as Decker's, Gouda, 1626. See *Phil. Mag.* Oct. 1872.

‡ The tables in this work were also published separately, I believe, under the title "A triangular Canon logarithmicall."

§ The proper date which should be assigned to Gunter's table, is of course, much earlier, before 1630; the date given above merely refers to the particular edition of Gunter's works that I used.

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however carefully performed, be of much value or interest. The tables included in the last-written list exhibit one very peculiar feature, which shows how each editor has blindly copied his predecessor, viz. in the log of 9482 which (correcting the seventh figure) is ·9769000; but it is printed as ·9768999 in Gunter, Oughtred 1657, Vlacq 1681 and 1757, and Donn 1789; although of course it is right in Vlacq 1628, and Briggs 1624, which extend to more decimals. This error, not being one of Vlacq's, is not counted in the numbers given in the list.

There is a great want of a convenient ten-figure table of the logarithms of numbers, as Vlacq and Vega are both scarce and inaccurate as printed; and I have some thoughts of reprinting and stereotyping the logarithms of numbers from Vlacq's *Arithmetica*, but whether I do so or not, there is no doubt that, some time or other, it will have to be done, and I therefore take the present opportunity of pointing out the importance of reprinting the table in Vlacq's, and not in Vega's, form. Vega is arranged like a seven-figure table, only as the logarithms having to be given to three figures more than in the latter, the columns headed by the units of the number occupy more than one page, the rest of the double page being occupied with the differences, placed all together: an arrangement exceedingly inconvenient, and liable to cause mistakes. It has the advantage of taking up only about half of the space occupied by Vlacq, but the extent of the latter (600 pp.) is not too great for a working table. I should like to see the portion of Vlacq from 10,000 to 100,000 reprinted exactly as given by him, without any abbreviation whatever, or alteration except that the differences should range with the upper of the two logarithms between which they are placed, that the characteristics should be omitted, catch-numbers and logarithms added at the top of each page, the figures of the *numbers* common to all the column given only at the top perhaps, and the whole printed in such type as to reduce the page to the size of a large octavo.

Mr. Sang's contemplated nine-figure million table will, if carried out, render the use of nine-figure logarithms nearly as convenient as that of seven-figure logarithms is now, except that a table in three volumes is not so handy as one in a single volume; but even with Mr. Sang's table, ten figures will not unfrequently be required in spite of the additional labour in obtaining the last figure.

While speaking about the arrangement of logarithmic tables, I may be permitted to mention that after examining a great number, it appears to me that Babbage's table, with a few improvements (viz. the separation by a small space of the three leading figures from the other figures of the block, the omission of the subscript points to denote the increase of the last figure, except when that figure is a 5, and the marking the change of the fourth figure of the logarithms in the middle of the line by an asterisk instead of a difference of type) would be the very best possible form. Babbage paid more attention to the mode of printing, the thickness of the

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rules, the size of the type,* &c. than any other editor, and in consequence has the most nearly approached perfection. Mr. Sang, in his recently published seven-place table, prints the numbers and logarithms in the same type, and separated by reversed commas, omits all rules, and uses an Arabic nokta (a mark resembling the diamond in a pack of cards) to denote the change in the fourth figure in the middle of the line: all of which he regards as improvements, though after some use of the table, they seem to me to be quite the contrary. The nokta (which was first used by Shortrede in his tables 1844 and 1849) seem very objectionable, and not only the worst of the notations that have been proposed, but worse than no notation at all. It denotes the change well enough when the first figure is a 0, but when it is a 1 or a 2 there is no mark; and the fact that the latter cases occur but rarely aggravates the evil. A notation that explains its meaning to the eye, without care, ninety-nine times out of a hundred, is more likely to cause error than one that requires care each time.

In a note to my paper in the May number of the *Notices*, I remarked a discrepancy in the dates of the second and third editions of Sherwin, which De Morgan gives as 1717 and 1742, while a copy I had seen bore date 1726. Since writing that note I have taken pains to search out and examine the different editions of Sherwin. The editions I have seen are 1705 (this date being printed in Arabic numerals on the title-page), 1706, Roman numerals, 1717, 1726; the third edition, revised by Gardiner, 1741, and 1742; the fourth edition 1761, and the fifth edition (by Samuel Clark), 1771, and 1772. It thus appears that it was not at all an uncommon thing (probably as the impression was being made up from time to time) to advance the date by one year. The first four dates we may distribute among the first two editions as we please; perhaps 1705, 1706, and 1717, are the dates of the first impression, and 1726 of the second. A side-by-side comparison of the books would show the difference between the editions, but I have only seen them separately, and in distant libraries.

With regard to Vlacq's *Arithmetica* referred to in my former communications, I may mention that I have since seen a French edition, Gouda, 1628. (See *Phil. Mag.*, October, 1872.)

I scarcely think it necessary to apologise for making a communication on the subject of logarithms to the Royal Astronomical Society, as, though the method of constructing them is part of the province of mathematics, the tables themselves may well be said to belong to the science in which they find their most useful application. In all the dark ages of astronomy, before it was separated from astrology, its connexion with the trigonometrical canon is one of the facts in its history which it is most pleasant to

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* I say nothing about the shape of the figures, a point on which I find it difficult to form a decided opinion.

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contemplate; and now that the science has taken its proper rank, I do not think many would wish to see it separated from its old companions, the tables.

Cambridge, Dec. 12, 1872.

Postscript.—In my paper, in the May number of the *Notices*, I expressed an opinion that certain facts "seemed to imply that Vlacq meditated a Dutch translation of his work [the *Arithmetica* of 1628] but that the tables intended for the purpose were bought and published by George Miller." That a Dutch edition was not only proposed, but actually published, is, I think, proved by the following extract from a letter from Briggs to Pell dated October 25, 1628. Speaking of the filling in of the gap in the *Arithmetica* of 1624 he proceeds, "But I am eased of that charge and care by one Adrian Vlacque, an Hollander, who hathe done all the whole 100 chiliades, and printed them in Latin, Dutche, and Frenche, 1000 bookes in these three languages, and hath sould them almost all." The letter is in the Birch MSS., and is printed on p. 55 of the *Letters on Scientific Subjects*, published by the Historical Society of Science, under the editorship of Mr. Halliwell. I intend to quote the whole passage in which the above sentence occurs, in a paper I hope shortly to communicate to the *Philosophical Magazine*, on the subject of Vlacq's and Decker's tables. In all probability, the Dutch copies did not sell, so that they were bought by Miller, and published with the English, in place of the Dutch introduction.

March 14, 1873.

On the Apparent Projection of Stars upon the Moon's Disk in Occultations. By John J. Plummer, Esq.

(*Communicated by Prof. A. S. Farrer, D.D.*)

Few phenomena have more constantly resisted a satisfactory scientific explanation than the well-authenticated though anomalous projection of the images of stars upon the lunar disk previous to occultation. In the *Memoirs of the Royal Astronomical Society*, vol. iii. the whole subject has been thoroughly investigated by Sir James South, and in a later volume (xxviii.) the Astronomer Royal has also given the question careful consideration. The former astronomer, after examining five several hypotheses in order to explain the origin of the phenomenon, is obliged to reject all of them, without assigning any other; and the Astronomer Royal, though more happy, does not appear to consider his solution entirely satisfactory. Indeed, it fails principally in those cases where the phenomenon has been seen at the dark limb, or where a